E & M 3.

(a) 3 points

$\mathcal{E} = -\frac{d\phi}{dt}$ 1 point

$\phi = \int \mathbf{B} \cdot d\mathbf{A}$ 1 point

$\mathcal{E} = -B\frac{dA}{dt}$

$\mathcal{E} = -B\ell v_0$ (+ or − acceptable) 1 point

(b) 5 points

$\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B}$ or $F = I\ell B$ 1 point

$I = \left|\frac{\mathcal{E}}{R}\right|$ 1 point

For using $\mathcal{E}$ calculated in part (a) 1 point

$F = \left|\frac{\mathcal{E}}{R}\right| \ell B = \frac{B\ell v_0}{R}\ell B$

$F = \frac{v_0 B^2 \ell^2}{R}$ (+ or − acceptable) 1 point

For indicating that the force is opposite the direction of the velocity (including minus sign in above expression is sufficient) 1 point

(c) 5 points

For an expression of Newton's second law:

$F = ma$ 1 point

Using expression for F from part (b), with a generic velocity:

$a = \frac{vB^2\ell^2}{mR}$ (+ or − acceptable) 1 point

For indicating that the acceleration is opposite the direction of the velocity (including minus sign above or in differential equation below is sufficient) 1 point

For a correct differential equation:

$\frac{dv}{dt} = -v\,\frac{B^2\ell^2}{mR}$ 1 point

$\frac{dv}{v} = -\frac{B^2\ell^2}{mR}\,dt$

$\ln v\Big|_{v_0}^{v} = -\frac{B^2\ell^2}{mR}\,t$

$\ln\frac{v}{v_0} = -\frac{B^2\ell^2}{mR}\,t$

$v = v_0 e^{-B^2\ell^2 t/mR}$ 1 point

(d) 2 points

From energy conservation, the resistor will eventually dissipate all the kinetic energy of the rod

$E_{diss} = \frac{1}{2}mv_0^2$ 2 points